Supergravity interacting with bosonic p-branes and local supersymmetry

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(FTUV/01-2112, IFIC/01-70; December 21, 2001)

We study the coupling of supergravity with a purely bosonic brane source (bosonic p–brane). The interaction, described by the sum of their respective actions, is self-consistent if the bosonic p–brane is the pure bosonic limit of a super–p–brane. In that case the dynamical system preserves 1/2 of the local supersymmetry characteristic of the 'free' supergravity.

PACS numbers: 11.30.Pb, 11.25.-w, 04.65.+e, 11.10Kk

I. INTRODUCTION

It is usually expected that the addition of a pure bosonic system to a supersymmetric one must produce a complete breaking of the supersymmetry. Nevertheless, it has been shown [1] that the coupled system of supergravity on the orbifold spacetime $M^9 \times [S^1/\mathbf{Z}_2]$ and *pure bosonic* branes *fixed* at the orbifold fixed 'points' preserves 1/2 of the local supersymmetry, while the other 1/2 components of its parameter vanish on the brane due to the orbifold projection (see [2,3] for $D=5$ models; most of the models [2,3,1] are for domain walls, $p=D-2$). These systems have been discussed in connection with the search for a supersymmetric generalization [4] of the Randall-Sundrum Brane World scenario [5].

In this paper we show that, in general, a *dynamical* bosonic brane source interacting with *dynamical* supergravity preserves 1/2 of the local supersymmetry δ_{ls} exhibited by the supergravity action if the bosonic p–brane can be regarded as the bosonic limit of a super–p–brane. This is still sufficient to preserve the selfconsistency of the gravitino interaction. Moreover, on the worldvolume the local supersymmetry parameter has the characteristic structure of the κ–symmetry transformation of the superbrane.

Our notation is close to that in [6]. The (non-scalar) physical fields of the D–dimensional supergravity multiplet (graviton $e_\mu{}^a(x)$, gravitino $\psi_\mu^{\underline{\alpha}}(x)$, antisymmetric rank q gauge field(s) $C_{\mu_1\ldots\mu_q}(x)$ and the spin connection field $w_\mu^{ab}(x)$) are given by differential forms on spacetime M^D

$$e^a(x) = dx^\mu e_\mu{}^a(x) \quad [a,\ \mu = 0,1,\ldots,(D-1)]\ , \tag{1.1}$$

$$C_q \equiv \frac{1}{q!} dx^{\mu_q} \wedge \ldots \wedge dx^{\mu_1} C_{\mu_1\ldots\mu_q}(x)\ , \tag{1.2}$$

$$e^{\underline{\alpha}}(x) = dx^\mu \psi_\mu^{\underline{\alpha}}(x) = e^a \psi_a^{\underline{\alpha}}(x)$$
$$[\underline{\alpha} = 1,\ldots,n\ , \qquad n = \dim(Spin(1,D-1))]\ , \tag{1.3}$$

$$w^{ab}(x) = dx^\mu w_\mu^{ab}(x) = -w^{ba}(x)\ . \tag{1.4}$$

For simplicity we will not address here the cases where the supergravity multiplet contains scalar fields as well as when the brane carries worldvolume gauge fields. Thus our basic examples are $N{=}1$, $D{=}3,4,11$ supergravity interacting with a bosonic p–brane.

II. ACTION FOR THE COUPLED SYSTEM OF SUPERGRAVITY AND A BOSONIC BRANE SOURCE

The coupled system action is given by the sum

$$S = S_{D,SG} + S_{D,p,0} \equiv \int_{M^D} \mathcal{L}_D + \int_{W^{p+1}} \hat{\mathcal{L}}_{p+1}\ , \tag{2.1}$$

where $S_{D,SG}$ and $S_{D,p,0}$ are the supergravity and *bosonic* (hence the subindex 0) p–brane actions. $\mathcal{L}_D$ is the Lagrangian D-form on M^D,

$$\mathcal{L}_D = d^D x L_{sg}(x) = \mathcal{L}_D^2 + \mathcal{L}_D^{3/2} + \mathcal{L}_D{}^{\leq 1}\ , \tag{2.2}$$

and $\hat{\mathcal{L}}_{p+1}$ is the Lagrangian $(p+1)$–form on the worldvolume $W^{p+1} \subset M^D$ with local coordinates $\xi^i = (\tau, \sigma^1, \ldots, \sigma^p)$,

$$\hat{\mathcal{L}}_{p+1} = d^{p+1}\xi\, L_{brane}(\xi) = \frac{1}{2(p+1)} * \hat{e}_a \wedge \hat{e}^a - \hat{C}_{p+1} \tag{2.3}$$

(*cf.* [9,10] and refs. therein); the hat will be used from now on to denote the dependence on $\xi \in W^{p+1}$. The forms on W^{p+1} in Eq. (2.3),

$$\hat{e}^a = d\hat{x}^\mu(\xi) e_\mu{}^a(\hat{x}) = d\xi^i \partial_i \hat{x}^\mu(\xi) e_\mu{}^a(\hat{x})\ , \tag{2.4}$$

$$\hat{C}_{p+1} = \frac{1}{(p+1)!} d\hat{x}^{\mu_{p+1}} \ldots d\hat{x}^{\mu_1} C_{\mu_1\ldots\mu_{p+1}}(\hat{x})\ , \tag{2.5}$$

are the pull–backs $\hat{\phi}^*(e^a)$, $\hat{\phi}^*(C_{p+1})$ of the vielbein (1.1) and gauge field (1.2) forms on spacetime by the map

$$\hat{\phi} : W^{p+1} \to M^D\ , \qquad \hat{\phi} : \xi^i \mapsto \hat{x}^\mu(\xi)\ . \tag{2.6}$$

When $p \neq q-1$, *i.e.* if there is no C_{p+1} in the supergravity multiplet (Eq. (1.2)), then $\hat{C}$ is also absent in (2.3) and $\hat{\mathcal{L}}_{p+1}$ reduces to the Nambu–Goto term

$$\frac{1}{2(p+1)} \; * \hat{e}_a \wedge \hat{e}^a = \frac{1}{2} \, d^{p+1}\xi \, \sqrt{|g|} \, , \qquad (2.7)$$

where $*$ is the Hodge operator for a $(p+1)$–dimensional space with induced worldvolume metric $g_{ij} \equiv \hat{e}_i^a \hat{e}_{ja}$ on W^{p+1}. For p=0 (massless bosonic particle) and, *e.g.*, D=4, 11, for which there is no C_1 in the supergravity multiplet, Eq. (2.3) reduces to

$$\begin{aligned} \hat{\mathcal{L}}_1 &= \tfrac{1}{2} l(\tau) \hat{e}^a \hat{e}^b_\tau \eta_{ab} \, , \qquad (2.8) \\ \hat{e}^a &= d\tau \partial_\tau \, \hat{x}^\mu(\tau) \, e_\mu^a(\hat{x}) \, \equiv d\tau \hat{e}^a_\tau(\tau) \, , \end{aligned}$$

where $l(\tau)$ is a Lagrange multiplier (or worldline einbein) and the star operator can be defined by $*\hat{e}_a := l(\tau) \hat{e}_{\tau a}$. Note that, as $\delta\sqrt{|g|} = \sqrt{|g|} g^{ij} \hat{e}_{ia} \delta \hat{e}^a_j$,

$$\delta \hat{\mathcal{L}}_{p+1} = \tfrac{1}{2} * \hat{e}_a \wedge \delta \hat{e}^a - \delta \hat{C}_{p+1} \, , \qquad (2.9)$$

$$\delta \hat{\mathcal{L}}_1 = l(\tau) \hat{e}_{\tau a} \, \delta \hat{e}^a + \, d\tau \tfrac{1}{2} \delta l(\tau) \hat{e}^a_\tau \hat{e}_{\tau a} \, . \qquad (2.10)$$

A. Pure supergravity action and equations of motion

The most important terms in the *supergravity Lagrangian form* (2.2) are

$$\mathcal{L}_D^2 = R^{ab} \wedge e_{ab}^{\wedge(D-2)} \, , \qquad (2.11)$$

$$\mathcal{L}_D^{3/2} = -\tfrac{2i}{3} \mathcal{D} e^{\underline{\alpha}} \wedge e^{\underline{\beta}} \wedge e_{abc}^{\wedge(D-3)} \; \Gamma^{abc}_{\underline{\alpha}\underline{\beta}} \, . \qquad (2.12)$$

Here $e_{a_1 \ldots a_q}^{\wedge(D-q)} \equiv \frac{1}{(D-q)!} \varepsilon_{a_1 \ldots a_q b_1 \ldots b_{D-q}} e^{b_1} \wedge \ldots \wedge e^{b_{D-q}}$,

$$R^{ab} = dw^{ab} - w^{ac} \wedge w_c{}^b \qquad (2.13)$$

is the curvature of the spin connection (1.4), and

$$\mathcal{D} e^{\underline{\alpha}} = T^{\underline{\alpha}} = de^{\underline{\alpha}} - e^{\underline{\beta}} \wedge w_{\underline{\beta}}{}^{\underline{\alpha}} \, , \qquad (2.14)$$

where $w_{\underline{\beta}}{}^{\underline{\alpha}} := \frac{1}{4} \, w^{ab} \, \Gamma_{ab}{}_{\underline{\beta}}{}^{\underline{\alpha}}$, is the gravitino field strength.

We prefer here the first order formalism where the spin connection (1.4) is considered as an independent variable and the equations of motion $\delta S_{D,SG}/\delta w^{ab} = 0$ determine the 'improved' constraint on the spacetime torsion T^a,

$$\begin{aligned} T^a + ie^{\underline{\alpha}} \wedge e^{\underline{\beta}} \Gamma^a_{\underline{\alpha}\underline{\beta}} = 0 \qquad (2.15) \\ (T^a := \mathcal{D} e^a = de^a - e^b \wedge w_b{}^a) \, , \end{aligned}$$

which expresses the spin connection through $e^a_\mu(x)$ and $\psi^{\underline{\alpha}}_\mu(x)$.

In low dimensions, $D = 3, 4$, the Lagrangian form contains only the two terms (2.11), (2.12). In higher dimensions $\mathcal{L}_D$ also includes, in particular, the kinetic term for the gauge fields (1.2). This term can also be written in first order form. To this end one introduces the auxiliary $(q+1)$–form (see [7])

$$F_{q+1} \equiv \tfrac{1}{(q+1)!} e^{a_{q+1}} \wedge \ldots \wedge e^{a_1} \, F_{a_1 \ldots a_{q+1}}(x) \, , \qquad (2.16)$$

where $F^{a_1 \ldots a_{q+1}}$ can be used as well to build the bosonic $(D-q-1)$–form

$$\mathcal{F}_{D-q-1} = e_{a_1 \ldots a_{q+1}}^{\wedge(D-q-1)} \, F^{a_1 \ldots a_{q+1}}(x) \, . \qquad (2.17)$$

Then, $\mathcal{L}_D{}^{\leq 1}$ in Eq. (2.2) is

$$\mathcal{L}_D{}^{\leq 1} = c(\mathcal{H}_{q+1} - \tfrac{1}{2} F_{q+1}) \wedge \mathcal{F}_{D-q-1} + \ldots \, , \qquad (2.18)$$

$$\mathcal{H}_{q+1} := dC_q - c_1 e^{\underline{\alpha}} \wedge e^{\underline{\epsilon}} \wedge \bar{\Gamma}^{(q-1)}{}_{\underline{\alpha}\underline{\epsilon}} \, , \qquad (2.19)$$

$$\bar{\Gamma}^{(k)}{}_{\underline{\alpha}\underline{\epsilon}} := \tfrac{1}{k!} e^{a_1} \wedge \ldots \wedge e^{a_k} \Gamma_{a_1 \ldots a_k \underline{\alpha}\underline{\epsilon}} \, , \qquad (2.20)$$

where c, c_1 are constants depending on D and q, $\mathcal{H}_{q+1}$ is the generalized field strength of C_q and the terms denoted by dots do not contain $F^{a_1 \ldots a_{q+1}}(x)$. The variations $\delta F^{a_1 \ldots a_{q+1}}$ and δC_{p+1} of the 'free' supergravity action $S_{D,SG} = \int_{M^D} (\mathcal{L}^2_D + \mathcal{L}_D^{3/2} + \mathcal{L}_D{}^{\leq 1})$ produce the first–order form of the free gauge field equations

$$\mathcal{H}_{q+1} - F_{q+1} = 0 \, , \qquad (2.21)$$

$$G_{(D-q)} \equiv d(e_{a_1 \ldots a_{q+1}}^{\wedge(D-q-1)} F^{a_1 \ldots a_{q+1}}) + \ldots = 0 \, , \qquad (2.22)$$

and $\delta e^{\underline{\alpha}}$ and δe^a provide the Rarita–Schwinger and Einstein equations

$$\Psi_{(D-1)\underline{\alpha}} := \tfrac{4i}{3} \mathcal{D} e^{\underline{\epsilon}} \wedge e_{abc}^{\wedge(D-3)} \; \Gamma^{abc}_{\underline{\epsilon}\underline{\alpha}} + \ldots = 0 \, , \qquad (2.23)$$

$$M_{(D-1)\; a} := R^{bc} \wedge e_{abc}^{\wedge(D-3)} + \ldots = 0 \, . \qquad (2.24)$$

In the above notation a generic variation of the supergravity action reads

$$\begin{aligned} \delta S_{D,SG} = &-(-1)^D \int_{M^D} M_{(D-1)a} \wedge \delta e^a + \\ &+ (-1)^D \int_{M^D} \Psi_{(D-1)\underline{\alpha}} \wedge \delta e^{\underline{\alpha}} + \\ &+ (-1)^D \int_{M^D} e_{abc}^{\wedge(D-3)} \wedge (T^c + ie^{\underline{\gamma}} \wedge e^{\underline{\delta}} \Gamma^c_{\underline{\gamma}\underline{\delta}}) \wedge \delta w^{ab} + \\ &+ c \int_{M^D} (\mathcal{H}_{q+1} - F_{q+1}) \wedge e_{a_1 \ldots a_{q+1}}^{\wedge(D-q-1)} \; \delta F^{a_1 \ldots a_{q+1}} + \\ &+ c(-1)^{Dp} \int_{M^D} G_{D-q} \wedge \delta C_q \, . \qquad (2.25) \end{aligned}$$

We note for future use that *e.g.*, the $(D-1)$–form $M_{(D-1)a}$ above is related with the usual expression for the equations of motion $\delta S/\delta e^a_\mu(x)$ by

$$\begin{aligned} -(-1)^D M_{(D-1)a} &= (dx)_\mu^{\wedge(D-1)} \tfrac{\delta S}{\delta e^a_\mu(x)} \qquad (2.26) \\ &\equiv -(-1)^D (dx)_\mu^{\wedge(D-1)} M_a^\mu \, , \end{aligned}$$

where $(dx)_\mu^{\wedge(D-1)} \wedge dx^\nu = d^D x \, \delta^\nu_\mu$. Also, its external covariant derivative $\mathcal{D} M_{(D-1)a}$ is the D–form

$$\begin{aligned} \mathcal{D} M_{(D-1)a} &= (dx)_\mu^{\wedge(D-1)} \wedge dx^\nu \mathcal{D}_\nu M_a^\mu \\ &= d^D x \mathcal{D}_\mu \tfrac{\delta S}{\delta e^a_\mu(x)} \, . \qquad (2.27) \end{aligned}$$

B. Local supersymmetry of the pure supergravity action and symmetry under spacetime general coordinate transformations

The 'free' supergravity action $S_{D,SG}$ possesses *local supersymmetry* δ_{ls} (see [7,8]),

$$\delta_{ls} x^\mu = 0 \ , \qquad (2.28)$$

$$\delta_{ls} e^a = -2i e^{\underline{\alpha}} \Gamma^a_{\underline{\alpha\epsilon}} \epsilon^{\underline{\epsilon}}(x) \ , \qquad (2.29)$$

$$\delta_{ls} e^{\underline{\alpha}} = \mathcal{D}\epsilon^{\underline{\alpha}}(x) + \epsilon^{\underline{\epsilon}}(x) \mathcal{M}_{1\underline{\epsilon}}{}^{\underline{\alpha}} \ , \qquad (2.30)$$

$$\delta_{ls} C_{p+1} = 2c_1 e^{\underline{\alpha}} \wedge \bar{\Gamma}^{(p)}{}_{\underline{\alpha\epsilon}}\, \epsilon^{\underline{\epsilon}}(x) \ , \qquad (2.31)$$

$$\delta_{ls} w^{ab} = \mathcal{W}^{ab}_{1\underline{\epsilon}} \epsilon^{\underline{\epsilon}}(x) \ , \ \delta_{ls} F^{a_1 \ldots a_{q+1}} = S^{a_1 \ldots a_{q+1}}_{\underline{\epsilon}} \epsilon^{\underline{\epsilon}}(x) \ , \quad (2.32)$$

where $\epsilon^{\underline{\alpha}}(x)$ is the local supersymmetry parameter, $S^{a_1 \ldots a_{q+1}}_{\underline{\epsilon}}(x)$ and the one–forms $\mathcal{M}_{1\underline{\epsilon}}{}^{\underline{\alpha}}$, $\mathcal{W}_{1\underline{\epsilon}}{}^{ab}$ are constructed from the fields of the supergravity multiplet (1.1)–(1.4) and the auxiliary fields $F_{a_1 \ldots a_{p+2}}(x)$.

Besides the local supersymmetry (2.28)– (2.32), the action is invariant under *spacetime general coordinate transformations* δ_{gc} (see Appendix A) parametrized by $t^\mu(x)$,

$$\delta_{gc} x^\mu := x^{\mu\prime} - x^\mu = t^\mu(x) \ , \qquad (2.33)$$

$$\delta_{gc} e^a := e^a(x') - e^a(x) = \qquad (2.34)$$
$$= \mathcal{D}t^a(x) + e^b t^c T_{cb}{}^a + e^b i_t w_b{}^a \ , \qquad etc.$$

If the local Lorentz invariance of the theory is used, we can define

$$\delta_{gc} e^a = \mathcal{D}t^a + e^b t^c T_{cb}{}^a \ , \qquad etc. \ ; \qquad (2.35)$$

see Appendix A and Eqs. (A.3)– (A.12) for the transformations of all the supermultiplet components.

Note that it is also practical (and equivalent; see Appendix A) to define the action of δ_{gc} by its 'variational version' (see [13]) $\tilde{\delta}_{gc}$,

$$\tilde{\delta}_{gc} x^\mu = 0 \ , \quad \tilde{\delta}_{gc} e^a_\mu := \mathcal{D}_\mu t^a(x) + e^b_\mu t^c T_{cb}{}^a \ , \quad etc. \ , \quad (2.36)$$

i.e. by *identifying* $\tilde{\delta}_{gc} e^a_\mu$, Eq. (A.13), with the components $(\delta_{gc} e^a)_\mu$ of the differential form $\delta_{gc} e^a$ in (2.33) without varying the spacetime coordinates x^μ. The possibility of repacing δ_{gc} in (2.33) by its equivalent 'variational version' $\tilde{\delta}_{gc}$ in (2.36) is associated with the independence of differential forms on the choice of local coordinates *i.e.*, with the diffeomorphism invariance of differential forms, $e'^a(x') = e^a(x)$ *etc.* We shall come back to this point in connection with the second Noether theorem below.

C. Local symmetries and Noether identities for the pure supergravity action

In the framework of the second Noether theorem, the local supersymmetry (2.28)– (2.32) and the general coordinate transformation symmetry $\tilde{\delta}_{gc}$ (2.36) are reflected, respectively, by the (Noether) identities (see [6])

$$\mathcal{N}_{(D-1)\underline{\alpha}} := \mathcal{D}\Psi_{(D-1)\underline{\alpha}} -$$
$$-2iM_{(D-1)a} \wedge e^{\underline{\epsilon}} \Gamma^a_{\underline{\alpha\epsilon}} + \ldots \equiv 0 \ , \qquad (2.37)$$

$$\mathcal{N}_{(D-1)a} := \mathcal{D}M_{(D-1)a} - \ldots \equiv 0 \ , \qquad (2.38)$$

where the terms in the $(D-1)$–forms $\mathcal{N}_{(D-1)\underline{\alpha}}$ and $\mathcal{N}_{(D-1)a}$ denoted by dots turn out to be proportional to the *l.h.* sides of Eqs. (2.15), (2.21)–(2.23), but *not* of the Einstein equation (2.24) (see Sec. IV). Eqs. (2.37), (2.38) can be directly derived from the definitions of Ψ and M in (2.23), (2.24) (see [7] for their complete expressions in D=11 and Appendix B for $D=4$ case). Alternatively, substituting δ_{ls} in (2.29)–(2.32) or $\tilde{\delta}_{gc}$ in (2.36) for the generic δ in (2.25) one finds that the coefficients of the arbitrary parameters $\epsilon^{\underline{\alpha}}(x)$ and $t^a(x)$ are proportional to $\mathcal{N}_{(D-1)\underline{\alpha}}$ and $\mathcal{N}_{(D-1)a}$ respectively. Thus the local symmetries $\delta_{ls} S = 0$ and $\tilde{\delta}_{gc} S = 0$ imply the identities (2.37) and (2.38) respectively, and *viceversa.* This may be also easily verified in a complete form in $D=4$, $N=1$ supergravity, where (2.37) and (2.38) are given by (B.2) and (B.3) respectively in Appendix B, and where the variation (2.25) has the form (B.4).

The Noether identity for the general coordinate symmetry in its δ_{gc} form, where all the variations are the result of the change $\delta_{gc} x^\mu$, Eq. (2.33), should state that the variation of the action with respect to the coordinates x^μ does not produce an independent equation of motion. Thus this Noether identity should read

$$\mathcal{N}_\mu := \delta S_{D,SG} / \delta x^\mu \equiv 0 \ . \qquad (2.39)$$

To verify that we indeed have the identity (2.39) in free supergravity, it is convenient to separate the generic variation δ in (2.25) into the coordinate variation and the variation δ' of the fields. Then, with the above notation, a generic variation of the action is split [14] as follows

$$\delta S_{D,SG} \quad = \int_{M^D} (d^D x \mathcal{N}_\mu \delta x^\mu -$$
$$- \int_{M^D} (-)^D M_{(D-1)a} \wedge dx^\mu \delta' e^a_\mu(x) + \ldots \qquad (2.40)$$

where the dots indicate terms containing the variations δ' of the gravitino, spin connection fields *etc.*

As the action is written in terms of differential forms, the generic variation (2.25) is expressed in terms of the generic variations of these forms, δe^a *etc.* One can split the generic variation of each form into the effect produced by the coordinate variation δx (which is given by the Lie derivative, *e.g.* $e^a(x+\delta x) - e^a(x) = \mathcal{D}(\delta x^\mu e^a_\mu) + e^b \delta x^\mu e^c_\mu T_{cb}{}^a$), and the field variation δ'. With such a splitting Eq. (2.25) reads

$$\delta S_{D,SG} = -(-)^D \int_{M^D} M_{(D-1)a} \wedge (\mathcal{D}(\delta x^\mu e^a_\mu) +$$
$$+ e^b \delta x^\mu e^c_\mu T_{cb}{}^a + \ dx^\mu \ \delta' e^a_\mu) + \ldots =$$
$$= (-)^D \int_{M^D} (\mathcal{D}M_{(D-1)a}\, e^a_\mu \ - e^b e^c_\mu T_{cb}{}^a + \ldots) \delta x^\mu -$$
$$- (-)^D \int_{M^D} M_{(D-1)a} \wedge dx^\mu \ \delta' e^a_\mu + \ldots \ . \qquad (2.41)$$

Comparing (2.40) with (2.41) one finds the expression for $\mathcal{N}_\mu := \delta S_{D,SG}/\delta x^\mu$,

$$d^D x\, \mathcal{N}_\mu = (-)^D (\mathcal{D}M_{(D-1)\,a} \; + \ldots)\, e^a_\mu \,, \qquad (2.42)$$

where the meaning of the dots is the same as in Eqs. (2.37), (2.38). Then the desired Noether identity $\mathcal{N}_\mu \equiv 0$ (Eq. (2.39)) for the general coordinate symmetry δ_{gc} in (2.33) follows immediately from Eq. (2.38) (Eq. (B.3) in Appendix B for $D=4$, $N=1$ supergravity).

Since $\tilde{\delta}_{gc}$ and δ_{gc} are equivalent forms of the same spacetime general coordinate transformations, it may seem strange to have two independent Noether identities expressing the same local invariance, Eqs. (2.38) and (2.39). However, one realizes that Eq. (2.42), written as

$$d^D x\, \mathcal{N}_\mu - (-)^D (\mathcal{D}M_{(D-1)\,a} \; + \ldots)\, e^a_\mu \;\; \equiv 0 \,, \qquad (2.43)$$

is also another Noether identity which reflects diffeomorphism invariance. Indeed, it implies that a combination of the equations of motion $\delta S_{D,SG}/\delta x^\mu$, $\delta S/\delta e^a_\mu(x)$, *etc.* and their derivatives, namely (see (2.27))

$$\frac{\delta S_{D,SG}}{\delta x^\mu} + \left(\mathcal{D}_\nu \frac{\delta S_{D,SG}}{e^a_\nu(x)} + \ldots \right)\, e^a_\mu(x) \;\equiv 0 \,, \qquad (2.44)$$

vanishes identically, and thus it may replace either (2.39) or (2.38) as an independent identity. The Noether identity (2.43) reflects the independence of any differential form on the choice of the local coordinates used to write it. This *diffeomorphism* invariance means that any variation δ_{diff} of coordinates

$$x^\mu \mapsto x^{\mu\prime} = x^\mu + \delta_{diff} x^\mu \qquad \delta_{diff} x^\mu = b^\mu(x) \qquad (2.45)$$

can be supplemented by the corresponding variation of the component functions of the differential forms, *e.g.* by a change of frame $e^a_\mu(x) \to e^{a\prime}_\mu(x) = e^a_\mu(x) - (L_b e^a)_\mu$ (*cf.* Eq. (2.36)),

$$\delta'_{diff} e^a_\mu(x) = -(L_b e^a(x))_\mu \,, \qquad (2.46)$$

in such a way that

$$\delta_{diff} e^a := e^{a\prime}(x') - e^a(x) = 0 \,. \qquad (2.47)$$

The variation of the action vanishes trivially for such transformations, $\delta_{diff} S_{D,SG} \equiv 0$ since δ_{diff} of any differential form vanishes. On the other hand, splitting $\delta_{diff} S_{D,SG}$ as in Eq. (2.40), one finds that $\delta_{diff} S_{D,SG} = \int_{M^D} (d^D x \;\; \mathcal{N}_\mu - (-)^D \mathcal{D}M_{(D-1)\,a} \;\; e^a_\mu + \ldots)\;\; b^\mu(x)$. Hence diffeomorphism symmetry and the Noether identity (2.43) imply each other.

This also makes explicit why the symmetry $\tilde{\delta}_{gc}$ (2.36) may be identified as the variational copy of the general coordinate transformations δ_{gc} (2.33): one can obtain $\tilde{\delta}_{gc}$ for the parameter $t^a(x) = t^\mu(x) e^a_\mu(x)$ as a combination of a general coordinate transformation δ_{gc} with parameter t^μ and a change of local coordinates δ_{diff} with parameter $b^\mu = -t^\mu$: $\tilde{\delta}_{gc}(t) = \delta_{gc}(t) + \delta_{diff}(b=-t)$. Similarly, the Noether identity $\mathcal{N}_{(D-1)a}$ for $\tilde{\delta}_{gc}$ is proportional to the difference between the one for δ_{diff}, Eq. (2.43), and $dx^D \mathcal{N}_{(D-1)\mu}\, e^a_\mu$ (Eq. (2.39)) for δ_{gc}.

III. EQUATIONS FOR THE COUPLED SUPERGRAVITY—BOSONIC BRANE SYSTEM

The variation of the coupled action (2.1) with respect to the bosonic vielbein δe^a produces the *Einstein equation with source*,

$$M_{(D-1)a} = J^{(p)}_{(D-1)a} \,, \qquad (3.1)$$

$$J^{(p)}_{(D-1)a} \equiv (dx)^{\wedge (D-1)}_\mu \int_{W^{p+1}} *\hat{e}_a \wedge d\hat{x}^\mu \delta^D(x-\hat{x}) \,, \qquad (3.2)$$

$$J^{(p=0)}_{(D-1)a} \equiv (dx)^{\wedge (D-1)}_\mu \int_{W^1} d\hat{x}^\mu l(\tau) \hat{e}_{\tau a}\, \delta^D(x-\hat{x}) \,. \qquad (3.3)$$

The $(p{+}1)$–form gauge field equation (2.22) evidently acquires a source from the p–*brane* 'Wess–Zumino term' $\hat{C}_{p+1}$ (2.3) provided the gauge field C_q with $q{=}p{+}1$ enters the supergravity multiplet

$$c\, G_{D-p-1} = j_{D-p-1} \,, \qquad (3.4)$$

$$j_{D-p-1} = (dx)^{\wedge (D-p-1)}_{\mu_1 \ldots \mu_{p+1}} \frac{(-1)^{p+1}}{(p+1)!} j^{\mu_1 \ldots \mu_{p+1}} \,, \qquad (3.5)$$

$$j^{\mu_1 \ldots \mu_{p+1}} = \int_{W^{p+1}} d\hat{x}^{\mu_1} \wedge \ldots \wedge d\hat{x}^{\mu_{p+1}} \delta^D(x-\hat{x}) \,. \qquad (3.6)$$

As w^{ab}, $F_{a_1 \ldots a_q}$ and the gravitino 1–form $\hat{e}^{\underline{\alpha}} = \hat{e}^{\underline{a}} \psi^{\underline{\alpha}}_{\underline{a}}(\hat{x})$ do not appear in the bosonic p-brane action, eqs. (2.15), (2.21) as well as the Rarita–Schwinger equation (2.23) do not acquire a source term from $\delta S_{D,p,0}$. Nevertheless, this does not mean that the gravitino is free. Indeed, the covariant derivative in Eq. (2.23) involves the spin connection which is defined through a vielbein that satisfies the Einstein equation with source (3.1).

IV. LOCAL SYMMETRIES OF SUPERGRAVITY INTERACTING WITH A BOSONIC BRANE

A. Local supersymmetry

The supergravity part of the coupled system is of course invariant under the local supersymmetry (2.29)–(2.32) due to the identity (2.37) (now no longer a Noether identity due to the source in Eq. (3.1)). Thus, to study the invariance of the coupled action (2.1) under δ_{ls} it is sufficient to look at the variation of its bosonic p–brane part, $\delta_{ls} S_{D,p,0}$.

Let us consider first the case of a *bosonic massless particle*, $p=0$. Then (see Eq. (2.8)),

$$\delta_{ls} S_{D,0,0} = \int_{W^1} (l(\tau) \hat{e}_{\tau a}\, \delta_{ls} \hat{e}^a + d\tau \tfrac{1}{2} \hat{e}_{\tau a} \hat{e}^a_\tau \, \delta_{ls} l(\tau)) =$$
$$= \int_{W^1} (-2il(\tau) \hat{e}_{\tau a} \hat{e}^{\underline{\alpha}} \Gamma^a_{\underline{\alpha\epsilon}} \epsilon^{\underline{\epsilon}} + d\tau \tfrac{1}{2} \hat{e}_{\tau a} \hat{e}^a_\tau \, \delta_{ls} l(\tau)) \,, \qquad (4.1)$$

so that $\delta_{ls} S_{D,0,0} = 0$ *iff*, on W^1,

$$\epsilon^{\underline{\epsilon}}(\hat{x}) = \hat{e}^a_\tau \Gamma_a^{\underline{\epsilon\kappa}} \kappa_{\underline{\kappa}}(\tau) \equiv \tilde{\Gamma}^{\underline{\epsilon\kappa}} \kappa_{\underline{\kappa}}(\tau) \ , \qquad (4.2)$$

$$\text{and} \qquad \delta_{ls} l(\tau) = 4i \hat{e}^{\underline{\alpha}}_\tau \kappa_{\underline{\alpha}}(\tau) \ , \qquad (4.3)$$

where $\kappa_{\underline{\alpha}}(\tau)$ is a fermionic spinor function on the worldline; $\delta_{ls} S_{D,0,0} = 0$ follows from the fact that, algebraically, $\tilde{\Gamma}^{\underline{\epsilon\kappa}} \tilde{\Gamma}_{\underline{\kappa\delta}} = \delta_{\underline{\delta}}^{\underline{\epsilon}} \hat{e}_{\tau a} \hat{e}^a_\tau$.

On the mass shell, where $\delta S / \delta l(\tau) = 0$ implies $\hat{e}_{\tau a} \hat{e}^a_\tau = 0$, we find that the parameter $\epsilon^{\underline{\epsilon}}(\hat{x})$ defined by (4.2) contains only $n/2$ nonzero components. Thus, $1/2$ of the local supersymmetry is broken on the worldline W^1.

It is worth stressing that the local supersymmetry acts on the pull–back of the *bosonic* vielbein $\hat{e}^a \equiv d\hat{x}^\mu(\tau) e^a_\mu(\hat{x})$ in the same way as the fermionic κ–symmetry transformation δ_κ of the *super*particle [12] in a supergravity background,

$$S_{D,0} = \int_{W^1} \tfrac{1}{2} l(\tau) \hat{E}^a \hat{E}^b_\tau \eta_{ab} \ ; \qquad (4.4)$$

$$\delta_\kappa S_{D,0} = 0 \qquad \text{for}$$

$$\delta_\kappa \hat{x}^\mu := \hat{E}^a_\tau \Gamma_a^{\underline{\alpha\epsilon}} \kappa_{\underline{\epsilon}}(\tau) E^\mu_{\underline{\alpha}}(\hat{x}, \hat{\theta}) \ , \qquad (4.5)$$

$$\delta_\kappa \hat{\theta}^\epsilon := \hat{E}^a_\tau \Gamma_a^{\underline{\alpha\epsilon}} \kappa_{\underline{\epsilon}}(\tau) E^\epsilon_{\underline{\alpha}}(\hat{x}, \hat{\theta}) \ , \qquad (4.6)$$

$$\delta_\kappa l(\tau) = 4i \hat{E}^{\underline{\alpha}}_\tau \kappa_{\underline{\alpha}}(\tau) \ , \qquad (4.7)$$

acts on the pull–back of the *super*vielbein [15],

$$\hat{E}^a = d\hat{x}^\mu E^a_\mu(\hat{x}, \hat{\theta}) + d\hat{\theta}^\alpha E^a_\alpha(\hat{x}, \hat{\theta}) \ , \qquad (4.8)$$

$$\hat{E}^{\underline{\alpha}} = d\hat{x}^\mu E^{\underline{\alpha}}_\mu(\hat{x}, \hat{\theta}) + d\hat{\theta}^\beta E^{\underline{\alpha}}_\beta(\hat{x}, \hat{\theta}) \ , \qquad (4.9)$$

where $\hat{\theta} = \hat{\theta}(\xi)$. In other words, $\delta_\kappa \hat{E}^a(\hat{x}, \hat{\theta} = 0) = \delta_{ls} \hat{e}^a$ with $\epsilon^{\underline{\epsilon}}(\hat{x})$ given by (4.2). Let us stress that in the usual treatment of κ–symmetry [9,10] super–p–branes are considered in a *superfield supergravity background* (hence without considering a supergravity action), *i.e.* by having the supervielbeins (4.8), (4.9) restricted by the *superspace constraints*. These are simply the equations that follow by extending ($e^a \mapsto E^a$, $e^{\underline{\alpha}} \mapsto E^{\underline{\alpha}}$) Eqs. (2.15), (2.21) to superspace (x^μ, θ^α).

A similar situation occurs for a $p > 0$ bosonic brane *which is the bosonic 'limit' ($\hat{\theta}^\alpha(\xi) = 0$) of a superbrane* (see also Appendix C). For instance, for the D=11 membrane Eq. (2.31) reads $\delta_{ls} C_3 = e^{\underline{\alpha}} \wedge \bar{\Gamma}^{(2)}_{\underline{\alpha\epsilon}} \epsilon^{\underline{\epsilon}}(x)$ and, hence,

$$\begin{aligned}\delta_{ls} S_{11,2,0} &= \int_{W^3} \tfrac{1}{2} * \hat{e}_a \wedge \delta_{ls} \hat{e}^a - \delta_{ls} \hat{C}_3 = \\ &= \int_{W^3} (-i * \hat{e}_a \wedge \hat{e}^{\underline{\alpha}} \Gamma^a_{\underline{\alpha\epsilon}} - \hat{e}^{\underline{\alpha}} \wedge \hat{\bar{\Gamma}}^{(2)}_{\underline{\alpha\epsilon}}) \epsilon^{\underline{\epsilon}}(\hat{x}) \\ &= -i \int_{W^3} *\hat{e}_a \wedge \hat{e}^{\underline{\alpha}} (\Gamma^a (I - \bar{\gamma}))_{\underline{\alpha\epsilon}} \epsilon^{\underline{\epsilon}}(\hat{x}) \ , \end{aligned} \qquad (4.10)$$

where

$$\bar{\gamma} \equiv \frac{i}{3! \sqrt{|g|}} \epsilon^{ijk} \hat{e}^a_i \hat{e}^b_j \hat{e}^c_k \Gamma_{abc} \quad (\mathrm{tr} \bar{\gamma} = 0 \ , \ \bar{\gamma}^2 = I) \ , \qquad (4.11)$$

may be recognized as the $\hat{\theta}(\xi)$=0 value of the matrix $\bar{\gamma}^S \equiv i/(3! \sqrt{|g|}) \ \epsilon^{ijk} \hat{E}^a_i \hat{E}^b_j \hat{E}^c_k \Gamma_{abc}$ appearing in the κ–symmetry projector $(I + \bar{\gamma}^S)$ of the *super*membrane [9] (M2–brane)

$$S_{11,2} = \int_{W^3} \hat{\mathcal{L}}_3 = \int_{W^3} \tfrac{1}{2\,3!} * \hat{E}_a \wedge \hat{E}^a - \hat{C}_3(\hat{x}, \hat{\theta}) \ . \qquad (4.12)$$

Thus, when a bosonic p–brane is the bosonic limit of a superbrane, the coupled system of supergravity and this *bosonic* brane possesses on W^{p+1} $1/2$ of the original local supersymmetry δ_{ls} (2.28)–(2.32) with a 'κ–like' parameter

$$p > 0 \ : \quad \epsilon^{\underline{\alpha}}(\hat{x}) = (I + \bar{\gamma})^{\underline{\alpha\epsilon}} \kappa_{\underline{\epsilon}}(\xi) \qquad (4.13)$$

(see (4.2) for p=0 where $(I + \bar{\gamma}) \to \tilde{\Gamma}$). If the bosonic brane action is not obtained by setting $\hat{\theta}(\xi) = 0$ in the action of a superbrane, then $\delta_{ls} S_{brane} = 0$ would imply $\hat{\epsilon}^{\underline{\alpha}}(\hat{x}) = 0$ rather than (4.13) since the projector $(I - \bar{\gamma})$ in (4.10) would be replaced by a non–singular matrix in general.

Out of the worldvolume, *i.e.* on M^D but not on W^{p+1}, the local supersymmetry is preserved completely.

B. General coordinate transformations

In the same manner one finds that in the coupled system the (variational copy of the) general coordinate symmetry (2.36) of pure (super)gravity is partially broken and that their preserved part resembles the (variational copy of the) worldvolume general coordinate transformations (reparametrization symmetry) of the brane when acting on the pull–back of the differential forms. For instance, for the $p = 0$ coupled system one finds that under (2.36) $\tilde{\delta}_{gc} S = \tilde{\delta}_{gc} S_{D,0,0}$ becomes (*cf.* Eq. (4.1))

$$\begin{aligned}\tilde{\delta}_{gc} S_{D,0,0} &= \int_{W^1} (l(\tau) \hat{e}_{\tau a} \tilde{\delta}_{gc} \hat{e}^a + d\tau \tfrac{1}{2} \hat{e}_{\tau a} \hat{e}^a_\tau \tilde{\delta}_{gc} l(\tau)) \\ &= \int_{W^1} (l(\tau) \hat{e}_{\tau a} \mathcal{D} t^a + l(\tau) \hat{e}^b_\tau \, t^c T_{cb}{}^a) + \\ &\quad + \int_{W^1} d\tau \tfrac{1}{2} \hat{e}_{\tau a} \hat{e}^a_\tau \, \tilde{\delta}_{gc} l(\tau)) \ , \end{aligned} \qquad (4.14)$$

Thus $\tilde{\delta}_{gc} S = 0$ requires that on W^1 $\hat{t}^a$ is expressed in terms of a single function $k(\tau)$

$$\hat{t}^a \equiv t^a(\hat{x}) = \hat{e}^a_\tau k(\tau) \ , \qquad (4.15)$$

and

$$\tilde{\delta}_{gc} l(\tau) = -l(\tau) \mathcal{D}_\tau k(\tau) + k(\tau) \mathcal{D}_\tau l(\tau) \ . \qquad (4.16)$$

In the general $p \geq 0$ case, the D–dimensional general coordinate symmetry on M^D, $\tilde{\delta}_{gc}$ in Eq. (2.36), is partially broken on W^{p+1} down to a $(p+1)$–dimensional invariance on W^{p+1} with

$$\hat{t}^a \equiv t^a(\hat{x}) = \hat{e}^a_i k^i(\tau) \ . \qquad (4.17)$$

Note that Eq. (2.36) with (4.17) implies

$$\tilde{\delta}_{gc} \hat{e}^a_i = \mathcal{D}_i (\hat{e}^a_j k^j(\tau)) + e^b_i \ e^c_j k^j \ T_{cb}{}^a \ , \quad etc. \ , \qquad (4.18)$$

which is identical to $\delta_{gc} \hat{e}^a_i$ produced by $\delta_{gc} x^\mu$ with $\delta_{gc} \hat{x}^\mu = k^i(\xi) \partial_i \hat{x}^\mu(\xi)$ on W^{p+1} (general coordinate transformations on the worldvolume).

V. SELFCONSISTENCY CONDITIONS FOR THE COUPLED SYSTEM AND FERMIONIC EQUATIONS FOR THE BOSONIC PARTICLE

The *superbrane* fermionic equations of motion for *e.g.*, p=0 (see (4.8), (4.9) for notation),

$$\hat{E}^{\underline{\epsilon}} \Gamma^a_{\underline{\alpha\epsilon}} \hat{E}_{\tau a} = 0 \; , \tag{5.1}$$

have a well defined $\hat{\theta}(\xi) \to 0$ limit, $\hat{e}^{\underline{\epsilon}} \Gamma^a_{\underline{\alpha\epsilon}} \hat{e}_{\tau a} = 0$. One may ask whether this limit is reproduced by the dynamical system under consideration. We show now that this is the case; this will turn out to be important in the next section to check the consistency of the interaction.

The particle equations of motion $(\delta S/\delta \hat{x}^\mu)\ e^a_\mu(\hat{x}) = 0$ and $\delta S/\delta l(\tau) = 0$ are of course purely bosonic

$$\mathcal{D}(l(\tau)\, \hat{e}_{\tau a}) + l(\tau)\, \hat{e}_{\tau b} \hat{e}^c T_{ca}{}^b(\hat{x}) = 0 \; , \tag{5.2}$$

$$\hat{e}^a_\tau \hat{e}_{\tau a} = 0 \; . \tag{5.3}$$

But besides Eqs. (5.2), (5.3), there is a nontrivial equation on W^1 produced by the selfconsistency condition for the equation of motion (2.23),

$$\mathcal{D}\Psi_{(D-1)\underline{\alpha}} = 0 \; . \tag{5.4}$$

When p=0 and the supergravity multiplet does not contain a vector field (as, *e.g.*, in $D = 3, 4, 11$), the gauge field equation (2.22), the Rarita–Schwinger one (2.23) and the geometric equations (2.15), (2.21) remain sourceless; the source appears only in (3.1) and, thus, the terms denoted by dots in (2.37) (and in (2.38)) vanish on shell. In the interacting case these are no longer Noether identies since the Einstein equation acquires a source term. But they are still identities algebraically satisfied by the explicit expressions of the terms appearing in their *l.h.* sides. Then, using (2.37), the selfconsistency condition (5.4) becomes $\mathcal{D}\Psi_{(D-1)\underline{\alpha}} = 2iM_{(D-1)a} \wedge e^{\underline{\epsilon}} \Gamma^a_{\underline{\alpha\epsilon}} = 0$ which, in the light of (3.1), implies that the particle current (3.3) satisfies the additional equation

$$J^{(p=0)}_{(D-1)a} \wedge e^{\underline{\beta}} \Gamma^a_{\underline{\alpha\beta}} = 0 \; . \tag{5.5}$$

Using the properties of the delta function one finds that Eq. (5.5) is equivalent to $\int_{W^1} d\tau l(\tau) \hat{e}^{\underline{\epsilon}}_\tau \hat{e}_{\tau a} \Gamma^a_{\underline{\epsilon\alpha}} \delta^D(x - \hat{x}) = 0$. After integration with a probe function this in turn implies the *fermionic* equation on W^1

$$\hat{e}^{\underline{\beta}} \Gamma^a_{\underline{\beta\alpha}} \hat{e}_{\tau a} = 0 \; , \tag{5.6}$$

which coincides with the result of setting $\hat{\theta}(\tau) = 0 = d\hat{\theta}(\tau)$ in the fermionic equations of motion (5.1) for a massless *superparticle* (4.4) moving in a *superfield supergravity background* (and, in particular, in flat superspace).

Similarly, one can find that the (bosonic) selfconsistency condition for Eq. (3.1),

$$\mathcal{D}(M_{(D-1)a} - J^{(p=0)}_{(D-1)a}) = 0 \; , \tag{5.7}$$

being considered together with the identity (2.38) (no longer a *Noether* identity) implies the current conservation

$$\mathcal{D}J^{(p=0)}_{(D-1)a} = 0 \; , \tag{5.8}$$

which produces the additional bosonic equation on W^1

$$\mathcal{D}(l(\tau)\hat{e}_{\tau a}) = 0 \; . \tag{5.9}$$

However, Eq. (5.9) becomes equivalent to Eq. (5.2) after Eq. (5.6) is taken into account. Indeed, Eq. (2.15) (which is not changed in the coupled system) implies (see (1.3)) $T_{cb}{}^a = -i\psi^{\underline{\alpha}}_c \Gamma^a_{\underline{\alpha\beta}} \psi^{\underline{\beta}}_b$. Thus the second term in Eq. (5.2), $\hat{e}_{\tau b} \hat{e}^c_\tau T_{ca}{}^b = -i\hat{e}^c_\tau \psi^{\underline{\alpha}}_c \Gamma^b_{\underline{\alpha\beta}} \hat{e}_{\tau b}\ \psi^{\underline{\beta}}_a \equiv -i\hat{e}^{\underline{\alpha}}_\tau \Gamma^b_{\underline{\alpha\beta}} \hat{e}_{\tau b}\ \psi^{\underline{\beta}}_a$, is the product of the *l.h.s* of Eq. (5.6) and the pull–back of the gravitino field. It vanishes when Eq. (5.6) holds, and Eq. (5.2) becomes (5.9).

VI. GRAVITINO INTERACTION AND ITS CONSISTENCY

It is well known that locally supersymmetric theories allow for a consistent interaction of the spin 3/2 fields, and that this remains true in supergravity theories with broken local supersymmetry [18] (*e.g.* super–Higgs effect [17]). We need to check whether the present breaking on W^{p+1} of 1/2 of the local supersymmetry of free supergravity does not result in an inconsistency for the interacting system of supergravity and the bosonic brane.

In 'free' supergravity the on–shell unwanted degrees of freedom of the gravitino field are removed by means of the local supersymmetry (2.30),

$$\delta_{ls} \psi^{\underline{\alpha}}_\mu = \mathcal{D}_\mu \epsilon^{\underline{\alpha}} + \ldots \; , \tag{6.1}$$

where the dots denote terms where the parameter $\epsilon^{\underline{\alpha}}$ enters without derivatives, but in a product with fields. Thus, in the weak field approximation, Eq. (6.1) reduces to

$$\delta_{ls} \psi^{\underline{\alpha}}_\mu = \partial_\mu \epsilon^{\underline{\alpha}} \quad \text{(weak field approximation)} \; , \tag{6.2}$$

obviously a gauge symmetry of the standard massless free Rarita–Schwinger equation

$$\Gamma^{abc}_{\underline{\alpha\beta}} \partial_b \psi^{\underline{\beta}}_c = 0 \quad \text{(weak field approximation)} \; . \tag{6.3}$$

The reduction of the degrees of freedom of $\psi^{\underline{\alpha}}_\mu$ can be done either by using the above local supersymmetry in a 'covariant' manner to fix the gauge $\Gamma^a_{\underline{\alpha\beta}} \psi^{\underline{\beta}}_a = 0$ (which then produces $\partial^a \psi^{\underline{\alpha}}_a = 0$), or in a noncovariant way by fixing first a Coulomb–like gauge $\Gamma^I_{\underline{\alpha\beta}} \psi^{\underline{\beta}}_I = 0$ (with *e.g.*,

$I = 1, \dots, (D-1)$) and then using the residual gauge invariance of this equation (that exists for $\epsilon^{\underline{\alpha}}$ satisfying $\Gamma^I_{\underline{\alpha\beta}} \partial_I \epsilon^{\underline{\beta}} = 0$) to fix $\psi_0^{\underline{\alpha}} = 0$. This can be made because using the free Rarita–Schwinger equation (*i.e.*, the weak field approximation) one finds that in the above gauge $\psi_0^{\underline{\alpha}}$ satisfies $\Gamma^I_{\underline{\alpha\beta}} \partial_I \psi_0^{\underline{\beta}} = 0$.

The splitting $\psi_a^{\underline{\beta}} = (\psi_0^{\underline{\beta}}, \psi_I^{\underline{\beta}})$ is obviously arbitrary and, in particular, one can replace $\psi_0^{\underline{\beta}}$ by $\hat{e}^a_\tau \psi_a^{\underline{\beta}}(\hat{x}) = \hat{\psi}_\tau^{\underline{\beta}}$ when the pull back $\hat{\psi}_a^{\underline{\beta}}$ on W^1 is considered. Now, since in the coupled system $\epsilon^{\underline{\alpha}}(\hat{x})$ is restricted on W^1 by Eq. (4.2) (or by $\epsilon^{\underline{\alpha}}(\hat{x}) = (I + \bar{\gamma})^{\underline{\alpha}}{}_{\underline{\epsilon}} \kappa^{\underline{\epsilon}}(\xi)$ in the $p > 0$ case) we have to check that there is still enough freedom left in $\epsilon^{\underline{\alpha}}(x)$ to use Eq. (6.1) (Eq. (6.2)) as in the free case. The key point is to notice that Eq. (4.2) on W^1 (as well as its $p > 0$ counterparts) does not restrict the pull–backs of the derivatives of the local supersymmetry parameter $\epsilon^{\underline{\alpha}}$ in the directions 'orthogonal' to the worldline (worldvolume). In other words, among $(\mathcal{D}_\mu \epsilon^{\underline{\alpha}})(\hat{x})$ only the combination $\partial_\tau \hat{x}^\mu (\mathcal{D}_\mu \epsilon^{\underline{\alpha}})(\hat{x}) = \mathcal{D}_\tau \hat{\epsilon}^{\underline{\alpha}} = \mathcal{D}_\tau (\hat{e}^a_\tau \Gamma_a^{\underline{\alpha\kappa}} \kappa_{\underline{\kappa}})$ is restricted on shell by the arguments following (4.2). Thus the only combination of the on–shell gravitino field components $\psi_\mu^{\underline{\alpha}}$ whose transformation rule (6.1) is restricted on W^1 is $\hat{\psi}_\tau^{\underline{\alpha}} \equiv \hat{e}^a_\tau \psi_a^{\underline{\alpha}}(\hat{x}) \equiv \partial_\tau \hat{x}^\mu \psi_\mu^{\underline{\alpha}}(\hat{x})$ for which the leading term is

$$\delta_{ls} \hat{\psi}_\tau^{\underline{\alpha}} = \mathcal{D}_\tau \hat{\epsilon}^{\underline{\alpha}} + \dots = \hat{e}^a_\tau \Gamma_a^{\underline{\alpha\kappa}} \mathcal{D}_\tau \kappa_{\underline{\kappa}} + \dots \qquad (6.4)$$

since $\delta_{ls} \partial_\tau x^\mu \psi_\mu^{\underline{\alpha}}(\hat{x}) = \partial_\tau x^\mu \delta_{ls} \psi_\mu^{\underline{\alpha}}(\hat{x})$ by Eq. (2.28). However, $\hat{\psi}_\tau^{\underline{\alpha}}$ is subjected to Eq. (5.6) which, since $\hat{e}^{\underline{\alpha}} \equiv d\tau \partial_\tau x^\mu \psi_\mu^{\underline{\alpha}}(\hat{x}) \equiv d\tau \hat{\psi}_\tau^{\underline{\alpha}}$ (Eq. (1.3)), can be written as

$$\hat{\psi}_\tau^{\underline{\alpha}} \Gamma^a_{\underline{\alpha\beta}} \, \hat{e}_{\tau a} = 0 \, . \qquad (6.5)$$

The general solution of Eq. (6.5), $\hat{\psi}_\tau^{\underline{\alpha}} = \hat{e}^a_\tau \Gamma_a^{\underline{\alpha\kappa}} \, \nu_{\underline{\kappa}}(\tau)$, can be gauged away, $\hat{\psi}_\tau^{\underline{\alpha}} = 0$, by taking $\mathcal{D}_\tau \kappa_\kappa = -\nu_{\underline{\kappa}}(\tau) + \dots$.

Hence, despite that there is less supersymmetry on W^1, $\psi_\mu^{\underline{\alpha}}$ does not get additional degrees of freedom on W^1 and the spin 3/2 field still has only transversal (2 in $D = 4$) polarizations in the coupled case: the fermionic equation (6.5) replaces the broken part of local supersymmetry in the role of removing the unwanted gravitino degrees of freedom on W^1. As a result, the gravitino interaction in the supergravity—bosonic brane system is consistent *iff* 1/2 of the original local supersymmetry is preserved on W^1. As we have seen in Sec. IV, this happens when the bosonic brane is the pure bosonic limit of a superbrane.

VII. ON THE MATCHING OF THE FERMIONIC AND BOSONIC DEGREES OF FREEDOM

We have concluded in Sec. IVA that the action for the supergravity—bosonic brane coupled system preserves 1/2 of the local supersymmetry δ_{ls} of the 'free' supergravity action. A faithful linear realization of supersymmetry requires equal number of bosonic and fermionic degrees of freedom. In this respect the above results could look surprising because they imply that local supersymmetry is still preserved (albeit partially) after adding a pure bosonic dynamical system, the p–brane, which presumably would introduce bosonic degrees of freedom on the worldvolume W^{p+1}.

A simple way of solving this paradox is to note that we are considering supergravity in the component formulation, where local supersymmetry acts on fields, but not on the spacetime coordinates (Eqs. (2.29)–(2.32), (2.28)). Thus, the brane coordinate functions $\hat{x}^\mu(\xi)$ are inert under local supersymmetry as well. In other words, $\hat{x}^\mu(\xi)$ are singlets (as, *e.g.* the heterotic fermions or chiral bosons in the heterotic superstring model), do not enter into the supergravity multiplet, and cannot produce a mismatch in the numbers of bosonic and fermionic degrees of freedom of such supermultiplet.

VIII. DIFFEOMORPHISM INVARIANCE AND P–BRANE DEGREES OF FREEDOM

Nevertheless, we may go beyond the above discussion by showing that in the supergravity–bosonic brane coupled system the bosonic p–brane does not carry any degrees of freeedom *i.e.*, that in the coupled system the p–brane degrees of freedom can be regarded as pure gauge. The key point is that the coupled action, for which the original general coordinate invariance of free supergravity is broken on W^{p+1} (Eq. (4.17)), is constructed in terms of differential forms and thus it is still *spacetime* diffeomorphism invariant, Eq. (2.47). This follows if the change of coordinates (2.45) is accompanied by

$$\delta_{diff} \hat{x}^\mu = b^\mu(\hat{x}) \, . \qquad (8.1)$$

for the p–brane coordinate functions $\hat{x}^\mu(\xi)$; then, on W^{p+1}, $\hat{e}^{a\prime}(\hat{x}') = \hat{e}^a(\hat{x})$. This local symmetry suggests that $(D - (p+1))$ brane degrees of freedom in $\hat{x}^\mu(\xi)$ may be regarded as 'pure gauge'. Indeed, whatever *e.g.* the massless particle worldline is, we can always use a general coordinate transformation to define (at least locally) a coordinate frame in which the worldline is represented by a (light–like) straight line. Similarly, for any worldvolume W^{p+1} it is possible to define local coordinates on M^D in such a way that W^{p+1} is described by $\hat{x}^{\mu\prime}(\xi) = (\xi^i, 0, \dots, 0)$ in the new coordinate system.

To justify that the bosonic diffeomorphism symmetry, Eqs. (2.45)–(2.47) and (8.1), is indeed a gauge symmetry of the coupled system (but *not* a *gauge* symmetry for the brane in (super)gravity background) one can use the second Noether theorem (see Sec. IIC). This implies the existence of a Noether Identity (NI) relating the *l.h.s.* of the equation $\delta S / \delta \hat{x}^\mu(\xi) = 0$ (Eq. (5.2) for $p = 0$) with the *l.h.* sides of the *field* equations of the coupled system, $\delta S / \delta e^a_\mu(x) = 0$ (Eq. (3.1)), $\delta S / \delta \psi_\mu^{\underline{\alpha}}(x) = 0$ (Eq. (2.23)), *etc.* The existence of such NI in the supergravity—bosonic brane coupled system has been actually proved at the end of Sec. V, where we have shown that the selfconsistency condition Eq. (5.7) for Eq. (3.1) produces Eq. (5.9) which, in turn, coincides with Eq. (5.2), after the selfconsistency condition (5.4) for Eq. (2.23) is taken into account. (Note that this NI is absent for the brane in a (super)gravity *background*, because such an approximation cannot produce the Einstein equation with a dynamical, *i.e.* $\hat{x}^\mu(\xi)$–dependent, source.)

Thus, we may conclude that *the bosonic brane does not carry any degrees of freedom in the coupled system* described by the sum of (super)gravity action and the brane action (see Appendix D for further discussion).

IX. FINAL REMARKS

The above suggests that the fermionic degrees of freedom of the superbrane in the supergravity—superbrane interacting system might be considered as pure gauge as well *i.e.*, that the superbrane degrees of freedom coupled to dynamical (*i.e.* not background) supergravity are pure gauge ones. Thus, one would expect that in the (singular) 'gauge' $\hat{\theta}(\xi) = 0$ any model for supergravity interacting with a dynamical superbrane source produces an action closely related (or equivalent) to (2.1). Indeed, the coupled system (2.1) is the result of 'fixing' the singular gauge $\hat{\theta}^\alpha(\xi)$=0 in the supergravity—super–p–brane coupled system described by the sum of the group manifold action for supergravity and the *superbrane* action [6].

This result is not so surprising as it might look at first sight. The possibility of gauging away the superstring degrees of freedom just reflects the fact that, by an appropriate choice of the coordinates, the brane may be located arbitraryly with respect to a coordinate system. This does not mean, however, that the coupled system is gauge equivalent to 'free' supergravity, since the source term in the Einstein equation (3.1) cannot be removed by a gauge transformation, although the freedom to choose arbitrary local coordinates may simplify the expression for the current (Eqs. (3.2), (3.3)).

This gauge character of the brane degrees of freedom in the presence of *dynamical* supergravity (*i.e.*, described by an action rather than being introduced as a background) might be looked at as a property 'dual' to the fact that (linearized) supergravity appears in the quantum states spectrum of a superbrane (superstring) in *flat* superspace, although it is not present independently.

We conclude by the following observation. If we considered our supergravity—bosonic p–brane system with $p = D-2$ on the orbifold spacetime $M^D = M^{D-1} \times [S^1/\mathbf{Z}_2]$ in an approximation where the $(D-2)$–brane is fixed at the orbifold fixed 'points', the $(D-1)$–hyperplanes, and the brane dynamics were ignored, we would arrive at a model of the type considered in refs. [2,1]. Then the observed explicit breaking of 1/2 of the local supersymmetry of 'free' supergravity by the bosonic brane would correspond to the vanishing, at the orbifold fixed 'points', of the part of the supersymmetry parameter which is odd under the $\mathbf{Z}_2$ projection, a characteristic of the models [2,1]. However, our analysis indicates that the (partial) supersymmetry preservation is not a specific property of models on the orbifold spacetime $M^D = M^{D-1} \times [S^1/\mathbf{Z}_2]$ but rather that it is inherent of any Lagrangian description of the supergravity—bosonic p–brane interacting system, for any p, provided (see Sec. IVA) that the p–brane is the $\hat{\theta}(\xi) = 0$ 'limit' of a super–p–brane.

Acknowledgments. The authors are grateful to Dmitri Sorokin and Mario Tonin for useful discussions at the early stages of this work. This work has been partially supported by the DGICYT research grant PB 96-0756, Ucrainian FFR (research project #383) and the Junta de Castilla y León (research grant C02/199). Two of us (I.B. and J.A.) thank the Erwin Schrödinger Institute for Mathematical Physics and the organizers of the *Mathematical Aspects of String Theory* program for their kind hospitality during the final stages of this work.

APPENDIX A:

On the formulation of spacetime general coordinate transformations

Eq. (2.34) reflects the action of a general coordinate transformation $x^\mu \to x^{\mu\prime} = x^\mu + t^\mu(x)$ on the vielbein one–form $e^a(x) = dx^\mu e^a_\mu(x)$; it is given by the Lie derivative $L_t e^a(x) := e^a(x') - e^a(x) \equiv e^a(x+t) - e^a(x) = dx^\nu (t^\mu \partial_\mu e^a_\nu + \partial_\nu t^\mu \; e^a_\mu)$. Note that since differential forms are invariant under diffeomorphisms *i.e.*, they can be expressed in any local coordinate system ($e^{a\prime}(x') = e^a(x)$, *etc.*, where the primes refer to the new coordinate system), the Lie derivative may also be expressed as $-L_t e^a(x) := e^{a\prime}(x) - e^a(x)$. In fact, $e^{a\prime}(x') - e^a(x) = e^{a\prime}(x') - e^a(x') + e^a(x') - e^a(x) = \delta'_{diff} e^a(x) + L_b e^a(x) = -L_b e^a(x') + L_b e^a(x) = 0$ since $L_b e^a(x') = L_b e^a(x)$ at first order.

Using that $L_t e^a(x) = (di_t + i_t d) e^a(x) = dt^a + i_t(de^a)$ we find

$$\delta_{gc} e^a(x) = L_t e^a(x) = \mathcal{D}t^a(x) + i_t T^a + e^b i_t w_b{}^a \; , \quad \text{(A.1)}$$

where $t^a := i_t e^a = t^\mu e^a_\mu$, $i_t T^a = e^b t^c T_{cb}{}^a$ and the last term $i_t w_b{}^a = t^c w_{cb}{}^a$ is a local Lorentz transformation induced by the local translations $t^\mu(x)$. Introducing the 'covariant' Lie derivative,

$$\mathcal{L}_t := i_t \mathcal{D} + \mathcal{D} i_t \; , \quad \text{(A.2)}$$

we see that $\mathcal{L}_t e^a = \mathcal{D} i_t e^a + i_t \mathcal{D} e^a$, where $\mathcal{D} e^a = T^a$. Thus $\delta_{gc} e^a(x)$ in (A.1) is the sum of $\mathcal{L}_t e^a$ and of the induced Lorentz rotation $e^b i_t w_b{}^a$, $\delta_{gc} e^a(x) = \mathcal{L}_t e^a + e^b i_t w_b{}^a$.

For the full supergravity multiplet components we have

$$\begin{aligned}\delta_{gc}e^a(x) &:= L_t e^a(x) = \mathcal{L}_t e^a(x) + e^b i_t w_b{}^a(x) = \\ &= \mathcal{D}t^a + i_t T^a + e^b i_t w_b{}^a \ , \qquad (A.3)\\ \delta_{gc}e^{\underline{\alpha}}(x) &:= e^{\underline{\alpha}}(x+t) - e^{\underline{\alpha}}(x) = L_t e^{\underline{\alpha}}(x) = \\ &= \mathcal{L}_t e^{\underline{\alpha}}(x) + e^{\underline{\beta}} i_t w_{\underline{\beta}}{}^{\underline{\alpha}}(x) \ , \qquad (A.4)\\ \delta_{gc}C_q(x) &:= C_q(x+t) - C_q(x) = L_t C_q(x) = \\ &= i_t dC_q + d(i_t C_q) \ , \qquad (A.5)\\ \delta_{gc}w^{ab}(x) &:= L_t w^{ab}(x) = i_t R^{ab} + \mathcal{D}(i_t w^{ab}) \ , \qquad (A.6)\end{aligned}$$

$$\begin{aligned}\delta_{gc}F^{a_1\ldots a_{q+1}}(x) &:= L_t F^{a_1\ldots a_{q+1}}(x) = \\ &= t^\mu \partial_\mu F^{a_1\ldots a_{q+1}}(x) = \\ &= i_t \mathcal{D}F^{a_1\ldots a_{q+1}}(x) + \\ &\quad + (q+1)F^{[a_1\ldots a_q|b} i_t w_b{}^{|a_{q+1}]}(x) \ , \qquad (A.7)\end{aligned}$$

where $i_t e^{\underline{\alpha}}(x) := t^\mu(x)\psi_\mu^{\underline{\alpha}}(x)$, $i_t w_b{}^a(x) := t^\mu w_{\mu b}{}^a(x)$ and $i_t w_{\underline{\beta}}{}^{\underline{\alpha}}(x) := t^\mu w_{\mu\underline{\beta}}{}^{\underline{\alpha}}(x) = (1/4) i_t w^{ab}\Gamma_{ab\underline{\beta}}{}^{\underline{\alpha}}$, $\mathcal{D}t^a = dt^a - t^b w_b{}^a$, $\mathcal{D}e^{\underline{\alpha}}(x) = de^{\underline{\alpha}}(x) - e^{\underline{\beta}} w_{\underline{\beta}}{}^{\underline{\alpha}}(x)$, $\mathcal{D}(i_t w^{ab}) = d(i_t w^{ab}) - (i_t w^{ac})\ w_c{}^b + (i_t w^{bc})\ w_c{}^a$ and $\mathcal{D}F^{a_1\ldots a_{q+1}}(x) = dF^{a_1\ldots a_{q+1}}(x) - (q+1)F^{[a_1\ldots a_q|b}(x) w_b{}^{|a_{q+1}]}(x)$.

In a theory with manifest local Lorentz symmetry the $i_t w_b{}^a$ terms in $\delta_{gc}e^a$, $\delta_{gc}e^{\underline{\alpha}}$, $\delta_{gc}w^{ab}$, $\delta_{gc}F^{a_1\ldots a_{q+1}}$ may be conveniently ignored, as well as the $d(i_t C_q)$ term in a theory with the gauge invariance $\delta_{gauge}C_q = d\Lambda_{q-1}$. In this case the general coordinate variations of the fields, Eqs. (A.3)–(A.7), reduce to

$$\delta_{gc}e^a(x) = \mathcal{D}t^a(x) + i_t T^a \ , \qquad (A.8)$$

$$\delta_{gc}e^{\underline{\alpha}}(x) = \mathcal{D}i_t e^{\underline{\alpha}}(x) + i_t \mathcal{D}e^{\underline{\alpha}} \ , \qquad (A.9)$$

$$\delta_{gc}C_q = i_t dC_q \ , \qquad (A.10)$$

$$\delta_{gc}w^{ab} = i_t R^{ab} \ , \qquad (A.11)$$

$$\delta_{gc}F^{a_1\ldots a_{q+1}}(x) = i_t \mathcal{D}F^{a_1\ldots a_{q+1}}(x) \ . \qquad (A.12)$$

The $D = 4$, $N = 1$ superspace counterparts of these transformations are called ‘supergauge transformations’ in [19].

The component functions $(L_t e^a(x))_\mu$, say, of the one–form $L_t e^a(x)$ are given by $(L_t e^a(x))_\mu = t^\nu \partial_\nu e^a_\mu + \partial_\mu t^\nu\, e^a_\nu$. Since $L_t(dx^\mu e^a_\mu(x)) = L_t(dx^\mu)\ e^a_\mu(x) + dx^\mu L_t e^a_\mu(x)$, we see that the term $\partial_\mu t^\nu\, e^a_\nu$ in $(L_t e^a(x))_\mu$ comes from $L_t(dx^\mu) = d(L_t x^\mu) = d(\delta x^\mu) = dt^\mu$. Thus, if we now *define* the variation of the component function $\tilde{\delta}_{gc}e^a_\mu(x)$ by $(L_t e^a)_\mu(x)$ above,

$$\tilde{\delta}_{gc}e^a_\mu(x) = \mathcal{D}_\mu t^a + e^b_\mu t^c T_{cb}{}^a \ , \qquad (A.13)$$

then there is no need to vary x^μ since the effect of its variation has been already taken into account in (A.13). Thus we may set $\tilde{\delta}_{gc}x^\mu = 0$ and use $\tilde{\delta}_{gc}$ given by Eq. (A.13) (Eq. (2.36)) as an equivalent definition of the general coordinate transformations δ_{gc} when varying an action constructed from differential forms. For the other supergravity fields $\tilde{\delta}_{gc}$ is given by

$$\tilde{\delta}_{gc}e^{\underline{\alpha}}_\mu(x) = \mathcal{D}_\mu i_t e^{\underline{\alpha}}(x) + e^b_\mu t^c(\mathcal{D}e^{\underline{\alpha}})_{cb} \ , \qquad (A.14)$$

$$\tilde{\delta}_{gc}w^{ab}_\mu = e^c_\mu t^d R_{dc}{}^{ab} \ , \qquad (A.15)$$

$$\tilde{\delta}_{gc}F^{a_1\ldots a_{q+1}}(x) = t^\mu \mathcal{D}_\mu F^{a_1\ldots a_{q+1}}(x) \ , \qquad (A.16)$$

to which one may add

$$\tilde{\delta}_{gc}C_{\mu_1\ldots\mu_q} = (q+1)t^\nu \partial_{[\nu}C_{\mu_1\ldots\mu_q]} \ , \qquad (A.17)$$

ignoring the induced gauge transformations $d(i_t C_q)$.

The variation $\tilde{\delta}_{gc}$ is the so–called ‘variational version’ [13] of the general coordinate transformations δ_{gc} that act on forms as in Eq. (2.34).

APPENDIX B:

Noether identities for $D = 4$, $N = 1$ supergravity

For $D = 4$ simple ($N = 1$) supergravity, where the supergravity action contains only the two terms (2.11), (2.12),

$$\begin{aligned}S_{4,SG} = &\int_{M^4} \tfrac{1}{2}\epsilon_{abcd}R^{ab}\wedge e^c \wedge e^d \ + \\ &+ \tfrac{2i}{3}\int_{M^4} \epsilon_{abcd}\mathcal{D}e^{\underline{\alpha}} \wedge e^{\underline{\beta}} \wedge e^a\ \Gamma^{bcd}_{\underline{\alpha}\underline{\beta}} \ . \qquad (B.1)\end{aligned}$$

The complete expression for the Noether identities (2.37), (2.38) is

$$\begin{aligned}\mathcal{N}_{4\underline{\alpha}} &\equiv \mathcal{D}\Psi_{3\underline{\alpha}} - 2iM_{3a}\wedge e^{\underline{\beta}}\Gamma^a_{\underline{\alpha}\underline{\beta}} - \qquad (B.2)\\ &\quad -\Gamma_{a\underline{\alpha}\underline{\beta}}\ \mathcal{D}e^{\underline{\beta}} \wedge (T^a + ie^{\underline{\gamma}}\wedge e^{\underline{\delta}}\Gamma^a_{\underline{\gamma}\underline{\delta}}) \equiv 0 \ ,\\ \mathcal{N}_{4a} &:= \mathcal{D}M_{3a} - \qquad (B.3)\\ &\quad -\frac{1}{2}\epsilon_{abcd}R^{bc}\wedge(T^d + ie^{\underline{\alpha}}\wedge e^{\underline{\beta}}\Gamma^d_{\underline{\alpha}\underline{\beta}}) \equiv 0 \ .\end{aligned}$$

The the *generic* variation of the action (B.1) is (*cf.* (2.25))

$$\begin{aligned}\delta S_{4,SG} = &-\int_{M^4} M_{3a}\wedge\delta e^a - \int_{M^4}\Psi_{\underline{\alpha}}\wedge\delta e^{\underline{\alpha}} \qquad (B.4)\\ &+ \tfrac{1}{3!}\int_{M^4}\epsilon_{abcd}e^a\wedge(T^b + ie^{\underline{\gamma}}\wedge e^{\underline{\delta}}\Gamma^b_{\underline{\gamma}\underline{\delta}})\wedge\delta w^{cd} \ .\end{aligned}$$

APPENDIX C:

Bosonic brane in the Polyakov–like formulation and local supersymmetry

The discussion in Sect. VIA of the $p = 0$ case involved the einbein as an initially independent field (see (2.8)). The analogue in the $p > 0$ cases requires treating also independently the worldvolume metric. This may be done by using the Brink–Di Vecchia–Howe–Polyakov [16] formulation,

$$\hat{\mathcal{L}}'_{p+1} = \frac{1}{4} * \hat{e}_a \wedge \hat{e}^a - \frac{(p-1)}{4}(-)^p * 1 - \hat{C}_{p+1} \ , \qquad (C.1)$$

where the Hodge star operator $*$ involves the auxiliary worldvolume metric $g_{ij}(\xi)$, $*\hat{e}_a \wedge \hat{e}^a =$

$d^{p+1}\xi\sqrt{|g|}g^{ij}\hat{e}_i^a\hat{e}_j^b\eta_{ab}$ and $(-)^p * 1 = d^{p+1}\xi\ \sqrt{|g|}$. The metric $g_{ij}(\xi)$ is determined by its equations of motion,

$$\frac{\delta S'_{D,p,0}}{\delta g^{ij}(\xi)} = 0 \quad \Rightarrow \quad g_{ij} = \hat{e}_i^a \hat{e}_j^b \eta_{ab} \ . \tag{C.2}$$

The δ_{ls} variation of S, with $S_{D,2,0}$ replaced by $S'_{D,2,0}$ for (C.1), contains, in addition to (4.10) (where now the $*$ is defined with an independent $g_{ij}(\xi)$), a term proportional to

$$\Delta_{ls} g^{ij}(g_{ij} - \hat{e}_i^a \hat{e}_{aj}) \ , \tag{C.3}$$

where

$$\Delta_{ls} g^{ij} \equiv \delta_{ls} g^{ij} - \tfrac{1}{2}\delta_{ls} g^{kl} g_{kl} g^{ij} - \\ - \frac{2}{\sqrt{|g|}} \hat{e}_k^{\underline{\alpha}} (\Gamma_{bc})_{\underline{\alpha}\underline{\beta}} \hat{e}^{\underline{\beta}} g^{k(i}\epsilon^{j)pq} \hat{e}_p^b \hat{e}_q^c \ , \tag{C.4}$$

and $\delta_{ls} g^{ij}$ is to be determined from $\delta_{ls} S'_{D,2,0} = 0$. The matrix $\bar{\gamma} \equiv \frac{i}{3!\sqrt{|g|}}\epsilon^{ijk}\hat{e}_i^a\hat{e}_j^b\hat{e}_k^c\Gamma_{abc}$ has now the properties $tr(\bar{\gamma}) = 0$, $\bar{\gamma}^2 = (\sqrt{|det(\hat{e}_i^a\hat{e}_{aj})|}/\sqrt{|g|})\,I$, where $g := \det g_{ij}$. On the mass shell $\sqrt{det(\hat{e}_i^a\hat{e}_{aj})} = \sqrt{|g|}$ due to Eq. (C.2), $\bar{\gamma}^2 = I$ and $(I-\bar{\gamma})$ is a projector. Hence, again $\delta_{ls} S'_{D,2,0} = 0$ for $\epsilon^{\underline{\epsilon}}(\hat{x}) = (I+\bar{\gamma})^{\underline{\alpha}\underline{\beta}}\kappa_{\underline{\beta}}(\xi)$, for which we can define $\delta_{ls} g_{ij}(\xi)$ in such a way that it compensates the contributions from the $\delta_{ls}\hat{e}^a$ and $\delta_{ls}\hat{C}_{p+1}$ variations.

Thus we conclude that the n parametric local supersymmetry is partially broken on W^{p+1} down to its $n/2$ parametric subgroup.

The above reasonings are in correspondence with the proof of the κ–symmetry of the supermembrane [9].

APPENDIX D:

Graviton degrees of freedom in the (super)gravity—p-brane coupled system

In Sec. VIII we have shown that, since the coupled (super)gravity—bosonic p–brane system possesses diffeomorphism invariance, Eqs. (2.45)–(2.47) with (8.1), the brane degrees of freedom are pure gauge.

One could ask, nevertheless, whether after fixing the gauge $\hat{x}^\mu(\xi) = (\xi^i, 0, \ldots, 0)$ the brane degrees of freedom could reappear somehow in the vielbein, in the sense that one would not have enough gauge freedom left to reduce the vielbein degrees of freedom on the worldvolume to the required $\frac{(D-2)(D-1)}{2} - 1$ ($= 2$ for $D = 4$) transversal ones. However, the degrees of freedom of the graviton $e_\mu^a(x)$ can be fixed by using general coordinate invariance, Eq. (2.33) or (2.36). Then, since the general coordinate transformations symmetry is partially broken on W^{p+1} in the coupled system, the question is whether there is enough general coordinate symmetry left to fix the degrees of freedom in e_μ^a on W^{p+1}.

An analysis similar to that presented above for the gravitino indicates that this is indeed possible. In 'free' supergravity, after removing from D^2 the $\binom{D}{2}$ antisymmetric degrees of freedom by local Lorentz invariance, the remaining unwanted $2D$ degrees of freedom of the vielbein field can be disposed of by making use of the general coordinate symmetry $\tilde{\delta}_{gc}$, Eq. (2.36). We now show that this can be done in the present supergravity–bosonic brane coupled system, despite that the components of the vector parameter t^a of $\tilde{\delta}_{gc}$, arbitrary out of W^{p+1}, are reduced to $(p+1)$ independent functions on W^{p+1} (see Eqs. (4.15), (4.17)).

To remove the (on–shell) unwanted $2D$ degrees of freedom of e_μ^a mentioned above, one uses the derivatives of t^a

$$\tilde{\delta}_{gc} e_\mu^a = \mathcal{D}_\mu t^a + \ldots \ , \tag{C.5}$$

(*cf.* (6.1)). This can be achieved fixing the gauge covariantly, $\partial^\mu e_\mu^a = 0$ (assuming the weak field approximation) and then using the residual 'd'Alembertian' gauge invariance ($D + D$ conditions), or by fixing a Columb–like gauge $\partial^I e_I^a = 0$ for the 'orthogonal' part and and then using its residual 'harmonic' gauge invariance to fix the 'tangential' part of the gauge $\partial^0 e_0^a = 0$ (again, $2D$ conditions).

As before, the separation (e_0^a, e_I^a) of components in e_μ^a is arbitrary. Our task now is to show that, in the interacting case, Eq. (C.5) can still be used to fix a counterpart of these gauges. The 'tangential' component of the derivative $(\mathcal{D}_\mu t^a)(\hat{x})$ along W^1 , $(\partial_\tau\hat{x}^\mu\mathcal{D}_\mu t^a)(\hat{x}) = \mathcal{D}_\tau\hat{t}^a = \mathcal{D}_\tau(\hat{e}_\tau^a k(\tau))$, depends on a single function $k(\tau)$ on W^1, while the 'orthogonal' components of $(\mathcal{D}_\mu t^a)(\hat{x})$ are unrestricted on W^1. Again, the only component of e_μ^a whose transformation rule (C.5) is restricted on W^1 is the pull–back $\hat{e}_\tau^a = \partial_\tau\hat{x}^\mu e_\mu^a(\hat{x})$ for which

$$\tilde{\delta}_{gc}\hat{e}_\tau^a = \mathcal{D}_\tau\hat{t}^a + \ldots = \hat{e}_\tau^a\ \mathcal{D}_\tau k(\tau) + \ldots \tag{C.6}$$

at leading order (*cf.* (6.4)) since $\tilde{\delta}_{gc}\partial_\tau\hat{x}^\mu e_\mu^a(\hat{x}) = \partial_\tau\hat{x}^\mu\tilde{\delta}_{gc}e_\mu^a(\hat{x})$ by Eq. (2.36). Thus the only danger of appearance of additional degrees of freedom (with respect to those in the 'free' supergravity case) comes from the 'tangential' part $\hat{e}_\tau^a$ for which, by (C.6), only one gauge function $k(\tau)$ is available. However, just this part is restricted on W^1 by Eq. (5.9),

$$\mathcal{D}_\tau(l(\tau)\hat{e}_\tau^a) = 0 \tag{C.7}$$

(*cf.* (6.5)). The worldline field $l(\tau)$ can be removed by the remaining freedom in $k(\tau)$ (more precisely, as (C.6) has basically the form of infinitesimal scale transformations of $\hat{e}_\tau^a$ with the parameter $(1 + \mathcal{D}_\tau k(\tau))$, we can choose $\mathcal{D}_\tau k(\tau)$ in such a way that the scaling results in $\hat{e}_\tau^a \mapsto l^{-1}\hat{e}_\tau^a$). Then, Eq. (C.7) becomes the counterpart of the gauge fixing condition $\partial_\tau\hat{e}_\tau^a + \ldots = 0$.

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